JOURNAL OF COMPUTATIONAL PHYSICS 3, 451–452 (1969)

NOTE

On the Application of Luke's Weight Coefficients for Evaluating the Linearized Rayleigh Problem in a Rarefied Gas Flow

The classical Rayleigh problem, which describes the flow field due to an impulsively started infinite flat plate in a fluid medium, has recently been studied satisfactorily by using the BGK kinetic equation. For the linearized problem, near and far field solutions are obtained analytically for short or long time intervals compared to the mean-collision time [1]. Cercignani has also given expressions for the velocity and shear stress at the plate surface for all time intervals [2]. However, numerical results in the entire flow field can be obtained for all time intervals, and this is accomplished by following a scheme described in Reference 3. This note indicates that the singular integral equation for the velocity can also be solved by applying Luke's weight coefficients near the logarithmic singularity [4]. The integral equation for the velocity is

$$\frac{q_x}{V} = \frac{\lambda t}{(\pi)^{1/2}} \int_0^\infty dw \frac{q_x}{V} \int_{|p-w|}^\infty \frac{dv}{v} \exp\left[-v^2 - \frac{\lambda t}{v} | p - w |\right] + \frac{1}{(\pi)^{1/2}} \int_p^\infty \exp\left[-v^2 - \frac{\lambda t}{v} p\right] dv, \tag{1}$$

where q_x is the velocity component in the direction of the plate motion and V is the plate velocity. The frequency of intermolecular collisions is λ, and t is the time. The distance normal to the plate is nondimensionalized by referring it to the distance traversed by the acoustic wave in time t, and this is denoted by p.

The singularity in Eq. (1) appears whenever $w = p$, and the kernel $K(| p - w |)$ behaves near the singularity, *for small values of* λt, as [3]

$$\lim_{p\to w} \left[K(| p - w |) + \frac{\lambda t}{(\pi)^{1/2}} \log| p - w |\right] \approx - \frac{\lambda t}{(\pi)^{1/2}} \frac{\gamma}{2}, \tag{2}$$

where γ is the Euler constant (0.5772157...). The singularity is confined in a small region by dividing the domain of integration into a number of subdivisions (appropriately for the application of a quadrature rule). In the present problem, the integration with respect to w is done by trapezoidal rule, and the interval is divided into 150 subdivisions. Choosing h for the length of a subinterval, Eq. (1) is written as

$$Q(p) = \int_0^{p-h} f(p, w)\, dw + \int_{p-h}^{p} f(p, w)\, dw + \int_p^{p+h} f(p, w)\, dw + \int_{p+h}^{\infty} f(p, w)\, dw + L(p), \tag{3}$$

where $Q(p) \equiv q_x(\lambda t, p)/V$. Only the second and third integrals involve the singularity, and the rest can easily be approximated by sums using the trapezoidal rule for integration.

The second integral I_2 can be written as

$$I_2 = \int_{p-h}^{p} \left[K(|p-w|) + \frac{\lambda t}{(\pi)^{1/2}} \log|p-w|\right] Q(w)\, dw - \frac{\lambda t}{(\pi)^{1/2}} \left[\int_0^h Q(p-w) \log \frac{w}{h}\, dw + \log \int_0^h Q(p-w)\, dw\right]. \tag{4}$$

The first integral in Eq. (4) can be replaced by a single trapezoid as the integrand has no singularity (see Eq. (2)). The second and third integrals are replaced by linear combinations of Qs' using, respectively, the Luke and Lagrange weight factors ([4], [5]). In a similar manner, the third integral in Eq. (3) is replaced by linear combinations of Qs'. Thus Eq. (1) is reduced to a set of simultaneous algebraic equations, and the solutions of these equations give the distribution of velocity in the flow field.

Results of computation are given in ARL 67-0124, Aerospace Research Laboratories, Wright–Patterson Air Force Base, Ohio.

References

1. Yoshio Sone, Kinetic theory analysis of linearized rayleigh problem. *Physics of Fluids.* 7, No. 3, March 1964.
2. Carlo Cercignani and Franco Sernagiotto, "Rayleigh's Problem at Low Mach Numbers According to Kinetic Theory," Ed. J. H. deLeeuw. [Proc. of the Fourth International Symposium on Rarefied Gas Dynamics]. Academic Press, New York, 1965.
3. K. S. Nagaraja, The linearized Rayleigh problem in a rarefied gas flow according to the BGK model. ARL 66-0186, Aerospace Research Laboratories, Wright–Patterson Air Force Base, Ohio.
4. Yudell L. Luke, Evaluation of an integral arising in numerical integration near a logarithmic singularity. *Math. Tables and Other Aids to Computation.* **10**, No. 53, January 1956.
5. Milton Abramowitz and Irene A. Stegun, "Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables," National Bureau of Standards, 1964.

K. S. Nagaraja
James P. Hudson

Wright-Patterson Air force Base
Ohio 45433

Printed in Belgium